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DEPARTMENT:- Chemical

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Assignment III

1 A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms.

b Exponential growth/decay (use of ODE)

ii Mixing problems.

2. $r = (t^2+3t)\hat{i} - 2\sin 3t\hat{j} + 3e^{2t}\hat{k}$

i $\frac{dr}{dt} = (2t+3)\hat{i} - 6\cos 3t\hat{j} + 6e^{2t}\hat{k}$

ii $\left.\frac{d^2r}{dt^2}\right|_{\cancel{t=0}} = 2\hat{i} + 18\sin 3t\hat{j} + 12e^{2t}\hat{k}$

iii $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2\hat{i} + 12\hat{k}$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2+12^2} = \sqrt{148} = 2\sqrt{37} = 12.17$

3 $A = x^2y\hat{i} + (xy+yz)\hat{j} + xz^2\hat{k}$

$B = yz\hat{i} - 3xz\hat{j} + 2xyk$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i $\nabla\phi = \frac{d\phi}{dx}i + \frac{d\phi}{dy}j + \frac{d\phi}{dz}k$

$\frac{d\phi}{dx} = 6xy + yz$ $\quad \frac{d\phi}{dy} = 3x^2 + xz - 8yz^2$

$\frac{d\phi}{dz} = xy - 8y^2z$

$\overset{x}{}\overset{y}{}\overset{z}{}$
At $(1, 2, 1)$

$\frac{d\phi}{dx} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{d\phi}{dy} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = -12$

$\frac{d\phi}{dz} = (1)(2) - 8(2)^2(1) = -30$

$\nabla\phi = 14\hat{i} - 12\hat{j} - 30\hat{k}$

ii $\nabla \cdot A = \frac{da_x}{dx} + \frac{da_y}{dy} + \frac{da_z}{dz}$

$A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\nabla \cdot A = 2xy + (x+2) + 2xz$

At $(1, 2, 1)$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2 = \underline{\underline{8}}$

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iii $\nabla \times B$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$$

$= \hat{i}(2x + 3x) - \hat{j}(2y - y) + \hat{k}(-3z - z)$

$= 5x\hat{i} - y\hat{j} - 4z\hat{k}$

At $(1, 2, 1)$

$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$

iv ~~gradient A~~ grad div A

grad $(2xy + (x+z) + 2xz)$

let div $A = C = \nabla A$

$\nabla(\nabla A) = \nabla C = \hat{i}\dfrac{\partial c}{\partial x} + \hat{j}\dfrac{\partial c}{\partial y} + \hat{k}\dfrac{\partial c}{\partial z}$

$= \hat{i}(2y + 1 + 2z) + \hat{j}(2x) + \hat{k}(1 + 2x)$

At $(1, 2, 1)$

$\nabla C = \hat{i}(2(2) + 1 + 2(1)) + \hat{j}(2(1)) + \hat{k}(1 + (2)(1))$

$= \hat{i}(4 + 1 + 2) + \hat{j}(2) + \hat{k}(1 + 2)$

$= 7\hat{i} + 2\hat{j} + 3\hat{k}$

v Curl Curl A

Curl $A = \nabla \times A$

$$= \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= \hat{i}(0-y) - \hat{j}(z^2-0) + \hat{k}(y-x^2)$$

$$= -y\hat{i} - z^2\hat{j} + \hat{k}(y-x^2)$$

At $(1, 2, 1)$

$$\text{Curl } A = -2\hat{i} - \hat{j} + \hat{k}$$

$$\text{Curl Curl} A = \nabla \times (\nabla \times A)$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= \hat{i}(1+2z) - \hat{j}(-2x^2-0) + \hat{k}(0+1)$$

$$= \hat{i}(1+2z) + 2x^2\hat{j} + \hat{k}$$

At points $(1, 2, 1)$

$$\nabla \times (\nabla \times A) = \hat{i}(1+2(1)) + 2(1)^2\hat{j} + \hat{k}$$

$$= 3\hat{i} + 2\hat{j} + \hat{k}$$